Whence, $\phi = (\pi/2) - (\theta/2)$. Also

$$R = \frac{[1 + (dy/dx)^2]^{-\frac{3}{2}}}{d^2y/dx^2} = 4a \sin \theta/2$$

Hence, $P = v_0^2 \div (4a \sin \theta/2) - g \sin \theta/2$. But, since $y = \theta(1 - \cos \theta)$,

$$2 \sin^2\frac{\theta}{2} = \frac{y}{a}\,.$$

Hence,

$$P = \frac{v_0^2 - 4ag \sin^2 \theta/2}{4a \sin \theta/2} = \frac{v_0^2 - 2gy}{4a \sin \theta/2},$$

But, at the point where the hound tumbled through, $y = y_0 = (v_0/2g)^2$. Hence, at that point, $P = 0$.

NUMBER THEORY.

212. Proposed by C. N. SCHMALL, New York City.

Given any positive integer N greater than 1; to prove that the sum of all the positive integers less than N and prime to N equals $\frac{1}{2}N \cdot \phi(N)$.

SOLUTION BY H. C. FEEMSTER, York College, Nebraska.

Let $N = a^h b^k c^l \cdots$, where $a, b, c, \cdots$ are primes. Then

$$\phi(N) = N \cdot \frac{a-1}{a} \cdot \frac{b-1}{b} \cdot \frac{c-1}{c} \cdot \cdots = N\left[1 - \Sigma\frac{1}{a} + \Sigma\frac{1}{ab} - \Sigma\frac{1}{abc} + \cdots\right],$$

the number of positive integers less than N and prime to N.

For every number $p < N/2$ and prime to N, there is a number $N - p > N/2$ and less than N and prime to N. Now there are $\phi(N)$ of these numbers or $\frac{1}{2}\phi(N)$ pairs of these numbers. But the sum of each pair is N, so the entire sum is $\frac{1}{2}N \cdot \phi(N)$, as required.

213. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that no relatively prime integers x and y exist such that the difference of their fourth powers is a perfect cube.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We are to prove the non-existence of any equation $x^4 - y^4 = z^3$. There are two cases to consider: (1) *x and y both odd*; (2) *one even and one odd.*

In the second case $x^2 + y^2$ and $x^2 - y^2$ are prime to each other, since any common factor would divide their sum $2x^2$, and difference $2y^2$; but these have, by hypothesis, 2 as their only common factor, and 2 is not a factor of $x^2 + y^2$ because x is even and y odd, or vice versa.

Since the product of $x^2 + y^2$ and $x^2 - y^2$ is a perfect cube, each of the factors must be, say, $x^2 + y^2 = a^3$, $x^2 - y^2 = b^3$. Call $2xy = c$. Then $a^6 - b^6 = c^2$, which is impossible. (See Number Theory, problem 209 in the March, 1914, MONTHLY which denies the existence of such an equation.)

Suppose now that x and y are both odd. Let $x = 4k \pm 1, y = 4l \pm 1$. Then $x^2 + y^2 = 16(k^2 + l^2) + 8(\pm k \pm l) + 2$, which is divisible by 2 but not by 4; and $x^2 - y^2 = 16(k^2 - l^2) + 8(\pm k \pm l)$, which is divisible by 8. Since $z^3 = (x^2 + y^2)(x^2 - y^2)$, it is divisible by 16 and hence by 64; and $x^2 - y^2$ by 32. Accordingly, we may write $x^2 + y^2 = 2\alpha^3$, $x^2 - y^2 = 32\beta^3$. We now see that if $x = 4k + 1$, $y = 4l + 1$, then $x + y$ is divisible by 2, but not by 4; any other combination of signs leads to this result for $x + y$ or $x - y$. We may then write, in the one case, $x + y = 2\gamma^3$, $x - y = 16\delta^3$. The other case may be similarly treated. Then

$$(x + y)^2 + (x - y)^2 = 4\gamma^6 + 256\delta^6 = 2x^2 + 2y^2 = 4\alpha^3.$$

Hence $\gamma^6 + 64\delta^6 = \alpha^3$, or $(\gamma^2)^3 + (2\delta^2)^3 = \alpha^3$, which is impossible. Hence there are no integral values of x and y *prime to each other* which satisfy the given equation.

Also solved by J. L. Riley

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell.

At the time of making up copy for this issue replies had not yet been received for numbers 4, 8, 11, 12, 13, 16, 18.

NEW QUESTIONS.

22. What can the Colleges do toward improving the teaching of mathematics in secondary schools?

23. What should be done with the theory of limits in elementary geometry? Should the recommendation of the National Committee of Fifteen on Geometry Syllabus be universally adopted? If not, what better disposition of the subject can be made?

REPLIES.

10. What use has been made of regular conference periods for assistance to individual students of secondary and college mathematics, and what services may they render?

Reply by Clyde S. Atchison, Washington and Jefferson College.

For the past two years, one hour each day has been set aside for conference in mathematics, during which time one member of the department is in his class room ready to assist students in settling their individual difficulties. The privilege thus offered has been much appreciated by a majority of the students, of whom many seize the opportunity for extra instruction to enlarge their grasp of the subject, while others, of less ability or unfortunate preparatory training, make use of the occasion to have explained to them those points to which only a limited amount of time can be devoted in the regular class work. A frank announcement in the class room, that no time will be wasted with a man who does not keep up with the work of the course, and that a conference hour is not a time